Automation, Stagnation, and the Implications of a Robot Tax

Emanuel Gasteiger[a,b] and Klaus Prettner[c]

a) Freie Universität Berlin,
School of Business & Economics,
Boltzmannstraße 20, 14195 Berlin, Germany
email: emanuel.gasteiger@fu-berlin.de

b) Instituto Universitário de Lisboa (ISCTE-IUL),
Business Research Unit (BRU-IUL),
Av. das Forças Armadas, 1649-026 Lisboa, Portugal

c) University of Hohenheim
Institute of Economics
Schloss, Osthof-West, 70593 Stuttgart, Germany
email: klaus.prettner@uni-hohenheim.de

Abstract

We assess the long-run growth effects of automation in the closed-form solution of the canonical overlapping generations framework. While automation implies constant returns to capital even in the absence of technological progress, positive long-run growth cannot emerge. This stands in sharp contrast to the representative agent setting with automation. The reason is that automation suppresses wages, which are the only source of investment in the canonical overlapping generations model. This result provides a cautionary tale that the underlying modeling structure of saving/investment decisions matters more in assessing the effects of automation than in the analysis of other phenomena in the growth literature. We show that a robot tax raises the capital stock and per capita output at the steady state. However, it cannot induce a takeoff toward positive long-run growth.

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1 Introduction

Automation and its potential economic consequences have caught the attention of economists, policymakers, and the general public over the last few years. For the recent breathtaking development of automation technologies, see, for example, The Economist (2014), Ford (2015), Brynjolfsson and McAfee (2016), and Tegmark (2017) who provide many examples for advances in robotics and artificial intelligence (AI) that were considered impossible even ten years ago. The number of industrial robots that substitute for workers on assembly lines started to take off in the 1990s (IFR, 2015) and 3D printing is already used to produce customized products like hearing aids and prostheses for which specialized labor was required in the past (Abeliansky et al., 2015). Currently, driverless cars and lorries that could soon revolutionize the employment-intensive transport business are being developed and tested. However, automation is not only confined to routine tasks: devices based on machine learning are starting to rival (and outcompete) doctors in the accuracy of diagnosing diseases, reporters in writing newsflashes, authors in writing books, and even scientists in formulating theories based on vast amounts of experimental data (see National Science Foundation, 2009; Schmidt and Lipson, 2009; Barrie, 2014; Ford, 2015).

On the one hand, there is widespread agreement that automation has a great potential to raise living standards (Steigum, 2011; Acemoglu and Restrepo, 2016; Graetz and Michaels, 2015; Hémous and Olsen, 2016; Abeliansky and Prettner, 2017; Eden and Gaggl, 2018; Prettner, 2018). On the other hand, there are also concerns that automation could (at least partly) be responsible for the stagnating wages of low-skilled workers, a phenomenon that we have observed in the United States since the 1970s (Frey and Osborne, 2013, 2017; Mishel et al., 2015; Arntz et al., 2016; Murray, 2016; Acemoglu and Restrepo, 2017; Dauth et al., 2017; Prettner and Strulik, 2017). Thus, automation might be a major driver of the rise in wage inequality and in the skill premium since the 1980s (Piketty and Saez, 2003; Piketty, 2014; Lankisch et al., 2017).

On top of these concerns, Sachs and Kotlikoff (2012), Benzell et al. (2015), and Sachs et al. (2015) argue that automation could lead to overall economic stagnation in the long run. They propose a model that, at the core, includes automation technology as a substitute for low-skilled workers in an overlapping generations economy. Sachs and Kotlikoff (2012) show analytically that advances in automation can reduce the wages of low-skilled workers depending on i) the substitutability between machines and labor in producing intermediate goods, ii) the substitutability between intermediate goods and skilled workers in producing the final good, and iii) the share of skilled labor in final output. Sachs and Kotlikoff (2012), Benzell et al. (2015), and Sachs et al. (2015) illustrate their main result of potential immiserizing productivity growth by relying on simulations of their model. Their results hint at the possibility that automation implies lower wages, lower investment, and a stagnating economy.

We aim to contribute to this strand of the literature along three lines. First, we show analytically that the long-run economic growth effects of automation crucially depend on the underlying framework that is used to describe the process of saving and investment. Since standard neoclassical growth models of Solow (1956), Cass (1965), Koopmans (1965), and Diamond (1965) lead to remarkably similar predictions with regards to the growth effects of capital accumulation, it is perhaps surprising that their implications regarding the growth effects of automation are diametrically opposed. Models of automation based on Solow (1956), Cass (1965), and Koopmans (1965), in which households save a part of their wage income *and* a part of their asset income, imply that automation could lead to perpetual long-run growth even without (exogenous or endogenous) technological progress. By contrast, models of automation based on the canonical overlapping generations (OLG) framework of Diamond (1965), in which households save *exclusively* out of wage income, imply economic stagnation in the face of automation. The reason for the differential effects of automation between the two types of underlying growth models is rooted in the demographic structure and the implied timing of events in the OLG model. The generation that builds up its assets for retirement can save *only* out of wage income. The resulting assets are in turn used to invest in standard physical capital and in automation. Since automation is, by its very definition, a substitute for labor, its accumulation suppresses wages and therefore diminishes the only source of investment in this model. As a result, automation is – in a sense – digging its own grave and preventing the takeoff to long-run growth in the OLG economy. This central result provides a cautionary tale that the underlying modeling structure of saving/investment decisions matters more in assessing the effects of automation than in the analysis of other phenomena in the growth literature. In particular, intergenerational frictions have the potential to counteract the positive effects of automation on growth. Nevertheless, our result should not be misinterpreted in the sense that we believe that economic stagnation is the likely outcome of progress in automation.

As a second contribution to the literature we analyze the effects of a robot tax coupled with a redistribution of the proceeds of the tax from robot income to labor income in such an OLG setting. We trace the effects of the tax-transfer scheme on the steady-state capital stock and thus on steady-state per capita output. We show that such a tax-transfer scheme cannot overcome the stagnation steady state and push the economy on a path with positive long-run growth. However, the tax-transfer scheme has a positive effect on the steady-state levels of per capita capital and per capita output. The intuitive explanation for this result is that the robot tax distorts the optimal split between investments in traditional physical capital and in automation capital in favor of the former. This raises the wages of the workers in the next period and allows them to save more. Thus, the traditional capital stock and automation capital in the next period are higher, which, in turn, raises aggregate income. The negative effect of the increase in automation capital on the wages of the workers in the next period, however, dampens the positive effect of the rise in the traditional capital stock to some extent. As a consequence, no permanent growth effect can emerge because of the robot tax but only a level effect.

Finally, from a more technical perspective, we show that the baseline model without the robot tax does not feature transitional dynamics, whereas in the model with a robot tax, transitional dynamics re-emerge. The reason for transitional dynamics to be present in case of a robot tax is rooted in the explanation for the positive level effect of the robot tax on income. The robot tax raises the wages of adults in the next period but the rise in automation capital dampens the effect of the higher capital stock on the wages so that they do not rise one-for-one with the aggregate capital stock as they would in a standard AK growth model. Instead, wages rise by less, such that convergence to a steady state takes place.

As far as the related literature on automation in the OLG model is concerned, we are only aware of the studies by Sachs and Kotlikoff (2012), Benzell et al. (2015), and Sachs et al. (2015). In contrast to these papers, we derive the closed-form analytical solution for the steady state in the canonical OLG model with automation. Thus, we are able to show analytically that automation sustains a negative feedback loop between wages and investment and to explain why this leads to different predictions than in the representative agent setting. Regarding the effects of a robot tax, a recent contribution by Guerreiro et al. (2018) analyzes the optimality of such a tax in a static model of production. In contrast to the standard result on production efficiency that it is not optimal to tax production inputs (Diamond and Mirrlees, 1971a,b; Judd, 1985; Chamley, 1986), Guerreiro et al. (2018) show how a robot tax increases welfare as long as automation is not yet full. Since their paper is concerned with the static efficiency gains of taxes on robots, whereas our paper is concerned with the effects of a robot tax on the dynamic forces that determine long-run growth in the OLG economy with automation, we view these two papers and their results as complementary.

The paper is structured as follows. In Section 2, we present the basic formulation of the OLG model with automation. In Section 3, we analyze the equilibrium dynamics and show that such a model necessarily leads to long-run stagnation. In Section 4, we show that the baseline mechanism is robust with respect to a three-period setting and with respect to exogenously evolving technology but not with respect to bequests. In section 5, we analyze the effects of a robot tax on the dynamics of the model and on the steady-state capital stock. In Section 6, we summarize and draw conclusions for policy makers.

2 Automation in the canonical OLG framework

Consider an economy in which time $t \in [0, 1 \ldots \infty)$ evolves discretely and households live for three time periods, youth, adulthood, and retirement. Children do not make any economic decisions and fulfill their needs via the consumption expenditures of their parents. Adults supply their available time on the labor market for the market clearing wage w_t and save for retirement. Retirees do not work and finance their consumption expenditures at old age out of their savings carried over from adulthood. The population growth rate is exogenous and denoted by $n > -1$ such that the evolution of the population size is given by $N_{t+1} = (1+n)N_t$, where N_t refers to the size of the adult cohort at time t.

Following Diamond (1965), households derive utility from consumption in adulthood, $c_{1,t}$, and from consumption in retirement, $c_{2,t+1}$. Assuming a logarithmic utility function to guarantee analytical tractability and that households discount the future at rate $\rho > 0$, which implies a discount factor of $\beta = 1/(1+\rho)$, the household's lifetime utility is given by

$$U_t = \log(c_{1,t}) + \beta \log(c_{2,t+1}). \tag{1}$$

Denoting the real interest rate on savings between time t and time $t+1$ by r_{t+1}, the budget constraint of households is standard and given by

$$c_{1,t} + \frac{c_{2,t+1}}{1 + r_{t+1}} = w_t, \tag{2}$$

where the left-hand side refers to discounted lifetime consumption expenditures and the right-hand side to lifetime labor income. Solving the households' intertemporal optimization problem yields the consumption Euler equation

$$\frac{c_{2,t+1}}{c_{1,t}} = \beta(1 + r_{t+1}) \tag{3}$$

describing the optimal consumption growth path for a given interest rate and a given discount factor. From this expression and the budget constraint, optimal consumption and savings of adults follow as

$$c_{1,t} = \frac{1}{1+\beta} w_t, \qquad s_t = \frac{\beta}{1+\beta} w_t. \tag{4}$$

Note that adults consume and save a fraction of their wage income in the first period, which allows them to build up assets for consumption when retired. However, young adults do not yet have any asset income that they could save, which stands in contrast to the models of Solow (1956), Cass (1965), and Koopmans (1965), where individuals start to accumulate assets at the first moment of their life.

While the consumption side is identical to the standard canonical OLG model, the production side changes in a fundamental way in the face of automation. There are now three production factors: labor, which is supplied by adults on the labor market, traditional physical capital in the form of machines, assembly lines, factory buildings, etc., which is an *imperfect* substitute for labor, and automation capital in the form of industrial robots, 3D printers, devices based on machine learning, etc., which is, by its very definition, a *perfect* substitute for labor (see, for example, the definition of automation in Merriam-Webster, 2017). When investing their savings, households can choose to buy traditional physical capital or automation capital.

The representative firm has access to a production technology as described by Prettner (2018)

$$Y_t = K_t^{\alpha}(N_t + P_t)^{1-\alpha}, \tag{5}$$

where Y_t denotes aggregate output (GDP), K_t denotes the stock of traditional physical capital, P_t denotes the stock of automation capital[1], and $\alpha \in (0,1)$ is the elasticity of output with respect to traditional physical capital. This production function conceptualizes the distinctive feature of automation capital as a perfect substitute for labor, which conforms to its very definition. Notice that the use of a more general production function such as in Steigum (2011), where the elasticity of substitution between labor and automation capital could be lower than in our case, would not deliver any additional insights in our context. The reason is that – with a lower elasticity of substitution – the potential for positive long-run growth would be reduced and for low enough levels it would not even emerge in the first place. in this case, we would be back in the standard well-known stagnation steady state, irrespective of whether we consider the models of Solow (1956), Cass (1965), and Koopmans (1965), or the model of Diamond (1965) as the baseline framework.

There is perfect competition in goods and factor markets such that all three production factors are paid their marginal value products. Using aggregate output as the numéraire, the profits of the representative firm are given by

$$\Pi_t = K_t^{\alpha}(N_t + P_t)^{1-\alpha} - w_t N_t - R_t^k K_t - R_t^p P_t, \tag{6}$$

where R_t^k is the rate of return on traditional physical capital and R_t^p is the rate of return on automation capital. The first term on the right-hand side is the revenue of the representative firm, whereas the last three terms are the costs of production in terms of the wage bill ($w_t N_t$), the expenses for traditional physical capital ($R_t^k K_t$), and the expenses for automation capital ($R_t^p P_t$). Profit maximization then implies the following factor rewards

$$w_t \overset{!}{=} R_t^p = (1-\alpha)\left(\frac{K_t}{N_t + P_t}\right)^{\alpha}, \tag{7}$$

$$R_t^k = \alpha\left(\frac{N_t + P_t}{K_t}\right)^{1-\alpha}. \tag{8}$$

Similar to the standard Diamond (1965) model, an increase in traditional physical capital raises the wage rate because it raises the machine intensity of the economy and therefore the productivity of workers. By contrast, an increase in automation capital has the opposite effect because automation capital competes closely with workers. Thus, an increase in the stock of automation capital does not raise

[1] The letter P stands for "programmable labor" because R for "robots" would lead to a confusion with the rate of return on capital.

worker's productivity as measured by their marginal value product but renders the workers more and more redundant. Notice, however, that labor productivity as measured by GDP per worker increases for both an increase in K_t and an increase in P_t. The reason is that an increase in both types of capital implies more production for a given amount of labor input.

3 Equilibrium and main results

For low levels of the traditional capital stock and for low levels of automation capital, Equations (7) and (8) imply

$$\lim_{P_t \to 0} R_t^p = (1-\alpha)\left(\frac{K_t}{N_t}\right)^{\alpha} \qquad \text{and} \qquad \lim_{K_t \to 0} R_t^k = \infty. \tag{9}$$

Consequently, the Inada conditions are not fulfilled for automation capital such that the possibility of a corner solution emerges. If the traditional capital stock and the automation capital stock are close to zero, individuals would only want to invest in the accumulation of physical capital because its return is higher. Only later, for a large enough traditional physical capital stock, an interior equilibrium on the capital market emerges. For certain parameters, investments in both types of capital then yield the same rate of return and individuals start to accumulate both, traditional physical capital and automation capital. Such an interior equilibrium of the capital market is characterized by a no-arbitrage relationship between both types of investment implying that $R_t^k = R_t^p$. From this condition, the following relationship between P_t and K_t emerges that holds in an interior capital market equilibrium

$$P_t = \left(\frac{1-\alpha}{\alpha}\right) K_t - N_t. \tag{10}$$

The intuition behind this relationship is best illustrated by referring to Equations (7) and (8): a higher stock of traditional physical capital (K_t) raises the rate of return on investment in automation capital (P_t) and reduces the rate of return on traditional physical capital. Hence, the stock of automation capital has to rise in response to re-establish the equality between the rates of return on traditional physical capital and on automation capital. By contrast, a larger cohort size of adults (N_t) implies that there are more workers available. In light of Equation (7), workers will have lower wages as a result such that the incentives for automation are lower. This reduces the incentives to invest in automation capital, which leads to a reduction in its equilibrium stock (see also Abeliansky and Prettner, 2017; Acemoglu and Restrepo, 2018, for theoretical considerations and for empirical support). Altogether, the behavior of the stock of automation capital is given by

$$P_t = \max\left\{0, \left(\frac{1-\alpha}{\alpha}\right) K_t - N_t\right\}, \tag{11}$$

which takes into account that households do not invest in automation capital if Equation (10) is negative. In this case the production function collapses to the standard expression in the canonical OLG model as given by $Y_t = K_t^\alpha N_t^{1-\alpha}$ and, consequently, the steady-state per capita capital stock and per capita income are constant.

To solve for the steady state that is associated with an interior equilibrium of the capital market, we plug the no-arbitrage relationship (10) into the production function (5). This yields an AK-type of technology in equilibrium

$$Y_t = \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} K_t, \tag{12}$$

where $A \equiv [(1-\alpha)/\alpha]^{1-\alpha}$. As is well-known, such a production structure usually leads to perpetual growth because there are constant returns with respect to the accumulation of physical capital (see, for example, Romer, 1986; Rebelo, 1991). As far as neoclassical models of automation that admit a representative household along the lines of Solow (1956), Cass (1965), and Koopmans (1965) are concerned, there is indeed the possibility of perpetual long-run growth for exactly this reason. For the theoretical derivation of these growth paths see Steigum (2011) and Prettner (2018). It is important to note that long-run growth is possible for a constant level of technology and it is *not* the result of knowledge spillovers due to a learning-by-doing mechanism. Instead, it follows directly from the feature of automation that it is a substitute for labor, which prevents the diminishing returns from capital accumulation from kicking in. Thus, the standard neoclassical convergence mechanism toward a steady state in which the economy stagnates is not operative in this setting.

As we show next, the fact that the OLG model with automation exhibits an AK-type of technology in case of an interior capital market equilibrium does not imply sustained growth. This stands in sharp contrast to the described findings of Steigum (2011) and Prettner (2018) for the representative agent neoclassical growth model with automation. Since the economy is closed and we follow the standard practice in OLG models by assuming that both types of capital fully depreciate over the course of one generation, the aggregate stock of assets at time $t+1$ is determined by investment in period t. This implies the following law of motion for the aggregate stock of assets

$$S_t = s_t N_t \stackrel{!}{=} K_{t+1} + P_{t+1} = \frac{\beta(1-\alpha)}{1+\beta} \left(\frac{K_t}{N_t + P_t} \right)^{\alpha} N_t. \tag{13}$$

We now define the competitive equilibrium of the economy with automation as follows.

Definition 1. *A competitive equilibrium is a sequence* $\{K_t, P_t, c_{1,t}, c_{2,t}, R_t, R_t^k, R_t^p, w_t\}_{t=0}^{\infty}$*, such that* $\{R_t,\ R_t^k, R_t^p, w_t\}_{t=0}^{\infty}$ *satisfy (7), (8), and* $R_t = R_t^k = R_t^p$*,* $\{c_{1,t}, c_{2,t}\}_{t=0}^{\infty}$ *satisfy (3) and (4),* $\{K_t, P_t\}_{t=0}^{\infty}$ *satisfy (10) and (13), and* $\{N_t\}_{t=0}^{\infty}$ *satisfies the population growth equation* $N_{t+1} = (1+n)N_t$*.*

Dividing Equation (13) by the size of the adult cohort N_{t+1} and plugging the no-arbitrage condition (10) into the result, we arrive at the capital accumulation equation

$$k = \alpha + \alpha \left(\frac{\beta}{1+\beta} \right) \left(\frac{1-\alpha}{1+n} \right) \left(\frac{\alpha}{1-\alpha} \right)^{\alpha}. \tag{14}$$

It is immediately clear that there are no transitional dynamics and that the steady-state capital-labor ratio of the economy is also given by $k_{t+1} = k_t = k$. Furthermore, there is no growth in the capital-labor ratio because the right-hand side of Equation (14) consists of constant parameters. From inspecting Equation (12) it follows that GDP per capita stagnates and there is no potential for long-run economic growth. We summarize our main finding on the long-run growth effects of automation in the canonical OLG economy in the following proposition.

Proposition 1. *In the canonical overlapping generations model with automation and an interior capital market equilibrium in which both traditional physical capital and automation capital are accumulated:*

i) the production structure resembles the properties of an AK type of growth model;

ii) the accumulation of automation capital reduces wages and therefore the savings/investments of households;

iii) the economy is trapped in a stagnation equilibrium because of the feedback effect between automation and wages.

This proposition implies that, in contrast to the standard neoclassical growth models with a representative agent, the economy necessarily stagnates in the canonical OLG model even if agents invest in both types of capital. The reason is that investment is fully financed out of wage income as implied by (4). However, wage income itself is reduced by automation. In a sense, automation is therefore digging its own grave in the OLG model. This result provides an analytical explanation for the numerical findings of Sachs and Kotlikoff (2012), Benzell et al. (2015), and Sachs et al. (2015) in the simplest possible OLG model that allows for closed-form solutions.

4 Extensions of the baseline model

In the following, we clarify the effects of some extensions of the model such as i) a three-period overlapping generations setting, ii) exogenous technological progress, and iii) bequests. We show that automation does not represent an additional engine of growth on top of technological progress in extensions i) and ii) for the very same reasons as described above. However, as we discuss below, bequests lead to a breakdown of the stagnation mechanism described above.

First, if we allow for a three-period overlapping generations structure, the consumption side of the economy changes, while the production side does not. Households then live for three periods and maximize

$$U_t = \log(c_{1,t}) + \beta \log(c_{2,t+1}) + \beta^2 \log(c_{3,t+2}) \tag{15}$$

subject to

$$c_{1,t} = w_t - s_t, \tag{16}$$

$$c_{2,t+1} = w_{t+1} + (1 + r_{t+1})s_t - s_{t+1}, \tag{17}$$

$$c_{3,t+2} = (1 + r_{t+2})s_{t+1}. \tag{18}$$

The resulting Euler equations are given by

$$\frac{c_{2,t+1}}{c_{1,t}} = \beta(1+r_{t+1}), \tag{19}$$
$$\frac{c_{3,t+2}}{c_{2,t+1}} = \beta(1+r_{t+2}). \tag{20}$$

Using the Euler equations and the budget constraints, we can derive savings as

$$s_t = \left[\frac{\beta+\beta^2}{(1+\beta+\beta^2)}\right] w_t - \left[\frac{1}{(1+\beta+\beta^2)}\right] \frac{w_{t+1}}{(1+r_{t+1})}. \tag{21}$$

Finally, from the no-arbitrage relation for investments in traditional physical capital and in automation capital (10), the equilibrium accumulation equation for the capital stock per worker follows as

$$\begin{aligned}\left(1+\frac{1-\alpha}{\alpha}\right) k_{t+1} = 1 &+ \left[\frac{(\beta+\beta^2)(1-\alpha)}{(1+\beta+\beta^2)(1+n)}\right]\left(\frac{\alpha}{1-\alpha}\right)^{\alpha} \\ &- \left[\frac{(1-\alpha)}{(1+\beta+\beta^2)(1+n)}\right] \frac{\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}}{\left[1+\alpha\left(\frac{\alpha}{1-\alpha}\right)^{\alpha-1}\right]}.\end{aligned} \tag{22}$$

It is immediately clear that the stagnation result from the baseline model carries over to the three-period overlapping generations case.

When we allow for labor-augmenting technological progress, the production side of the economy changes, while the consumption side does not. The modified production function is given by

$$Y_t = K_t^{\alpha}(A_t N_t + P_t)^{1-\alpha}, \tag{23}$$

where technology evolves according to $A_{t+1} = (1+g)A_t$ with $A_0 = 1$ and $g > 0$. The effective capital-labor ratio $\tilde{k} \equiv K/AL$ is given by

$$\tilde{k} = \alpha + \alpha\left(\frac{\beta}{1+\beta}\right)\frac{(1-\alpha)}{(1+n)(1+g)}\left(\frac{\alpha}{1-\alpha}\right)^{\alpha} \tag{24}$$

at the steady state. In this case, per capita variables grow along a balanced growth path at the rate of technological progress, g. For $g = 0$, Equation (24) collapses to Equation (14) and the economy is back in the stagnation steady state. Thus, with positive long-run growth, our result still holds true in the sense that automation does not represent an additional engine for long-run economic growth besides technological progress.

Finally, the possibility of bequests breaks the mechanism in the baseline model. This can be seen most easily by considering a multi-period discrete-time overlapping generations model with full bequests. In this case, the solution of the model converges to the one of Ramsey (1928), Cass (1965), and Koopmans (1965). In this case, long-run growth driven by automation would become possible again.

5 The effects of a robot tax

A natural question that emerges in our context is the extent to which redistribution policies can affect the impact of automation on the economy. In particular, a tax on robots is often suggested as a solution to mitigate some of the negative consequences of automation. For example, Bill Gates stated in an interview in 2017 that "[...]taxation is certainly a better way to handle it than just banning some elements of it." Gates also mentions how such a tax could be designed: "Some of it can come on the profits that are generated by the labor-saving efficiency there. Some of it can come directly in some type of robot tax." (Delaney, 2017). Furthermore, some governments and even the European Parliament are ventilating ideas about a robot tax (see, for example Prodhan, 2017). In the context of our model, it might be straightforward to conjecture that a tax on the income generated by robots and an associated redistribution of the proceeds of the tax toward workers who do not own assets could raise aggregate savings and enable the asset-poor parts of the population to share in the gains that automation brings about. While we show in the following that such a scheme is not effective in overcoming stagnation, the *level* of per capita income can be affected in case of the steady state with automation.

To conceptualize the tax-transfer scheme, we examine lump-sum transfers to the working age adults denoted by $\bar{\tau}_t$, which are financed by a tax on the use of automation capital by firms (the robot tax) at rate $\tau \in [0, 1]$. The budget constraint of households in the model with taxes and redistribution has to be modified and is given by

$$c_{1,t} + \frac{c_{2,t+1}}{1 + r_{t+1}} = w_t + \bar{\tau}_t, \tag{25}$$

where the lump-sum redistribution adds to the wage rate. The solution of the modified intertemporal optimization problem implies optimal consumption and savings of adults as

$$c_{1,t} = \frac{1}{1+\beta}(w_t + \bar{\tau}_t), \qquad s_t = \frac{\beta}{1+\beta}(w_t + \bar{\tau}_t). \tag{26}$$

The profit function of the representative firm in case of the tax-subsidy scheme becomes

$$\Pi_t = K_t^{\alpha}(N_t + P_t)^{1-\alpha} - w_t N_t - R_t^k K_t - (1+\tau) R_t^p P_t, \tag{27}$$

which takes into account that a robot tax increases the expenses of the employment of robots versus other types of machines. As a consequence, τ distorts the no-arbitrage condition between using traditional physical capital K_t and automation capital P_t in favor of using traditional capital K_t.

The lump-sum transfers to each adult are then given by

$$\bar{\tau}_t = \tau R_t^p \left(\frac{P_t}{N_t}\right). \tag{28}$$

With these ingredients, we can derive the capital accumulation equation as

$$\begin{aligned} k_{t+1} = {} & \frac{\alpha(1+\tau)}{1+\alpha\tau} \left\{ 1 + \frac{\beta(1-\alpha)}{(1+\beta)(1+n)} \left[\left(\frac{\alpha}{1-\alpha}\right)(1+\tau) \right]^{\alpha} \right. \\ & \left. \times \left[\frac{1}{(1+\tau)} + \frac{1}{(1+\tau)} \frac{(1-\alpha)}{\alpha(1+\tau)} k_t \right] \right\}. \end{aligned} \tag{29}$$

Setting $k_{t+1} = k_t = k$, the steady-state per capita capital stock in case of the tax-transfer scheme is given by

$$k^{\tau} = \frac{\alpha \left\{ (1+\beta)(1+n)(1+\tau) + (1-\alpha)\beta \left[\frac{\alpha(1+\tau)}{1-\alpha}\right]^{\alpha} \right\}}{\left\{ (1+\beta)(1+n)(1+\alpha\tau) - \alpha^2 \beta \tau (1+\tau) \left[\frac{\alpha(1+\tau)}{1-\alpha}\right]^{\alpha-2} \right\}}. \tag{30}$$

For $\tau = 0$, Equation (30) collapses to the steady-state per capita capital stock of the original model as given by Equation (14). We can now state the following result with respect to the effects of the robot tax-subsidy scheme.

Proposition 2. *In the canonical overlapping generations model with automation and an interior capital market equilibrium in which both traditional physical capital and automation capital are accumulated:*

i) a robot tax is not effective in overcoming stagnation;

ii) a robot tax raises per capita capital and thereby per capita income at the steady state.

Proof. Part i) of the proposition follows immediately from inspecting Equation (30) and noting that the right-hand side is constant.

For part ii), note that the derivative of the steady-state per capita capital stock with respect to the robot tax rate is given by

$$\begin{aligned}\frac{\partial k^{\tau}}{\partial \tau} = & \frac{(1-\alpha)\alpha\left\{(1-\alpha)\beta(1+\beta)(1+n)\left(1-\tau^{2}\right)\left[\frac{\alpha(1+\tau)}{1-\alpha}\right]^{\alpha}\right\}}{\left\{(\alpha-1)^{2}\beta\tau\left[\frac{\alpha(1+\tau)}{1-\alpha}\right]^{\alpha}-(1+\beta)(1+n)\left(1+\tau\right)\left(1+\alpha\tau\right)\right\}^{2}} \\ & + \frac{(1-\alpha)\alpha\left\{(1+\beta)^{2}(1+n)^{2}\left(1+\tau\right)^{2}+(\alpha-1)^{2}\beta^{2}\left[\frac{\alpha(1+\tau)}{1-\alpha}\right]^{2\alpha}\right\}}{\left\{(\alpha-1)^{2}\beta\tau\left[\frac{\alpha(1+\tau)}{1-\alpha}\right]^{\alpha}-(1+\beta)(1+n)\left(1+\tau\right)\left(1+\alpha\tau\right)\right\}^{2}} > 0.\end{aligned} \tag{31}$$

Since α and τ are both between zero and one, the numerator in both terms on the right-hand side is positive. From the fact that the denominator is squared, it then follows that the whole derivative is always positive. Consequently, the robot tax raises per capita capital and per capita output at the steady state. □

The intuitive explanation for this result is that the robot tax distorts the optimal split between investments in traditional physical capital and in automation capital in favor of the former. This raises the wages of the workers in the next period and allows them to save and invest more. Thus, the aggregate capital stock is higher in the next period, which leads to a higher aggregate income. The negative effect of the increase in automation capital on the wages of the workers in the next period dampens the positive effect of the rise in the traditional capital stock to some extent. As a consequence, no permanent growth effect can emerge.

Altogether, the robot tax has the potential to raise per capita capital and per capita output at the steady state of the canonical OLG model with automation. However, this result is only derived for a closed economy, where capital in either form cannot move abroad. In an open economy setting, the robot tax faces the same difficulty as a tax on financial transactions (the "Tobin Tax") in the sense that it is very easy to move a mobile production factor to a jurisdiction that does not impose such a tax. A successful implementation of a robot tax then depends on whether or not it is implemented by many countries jointly. Thus, the results of our model with respect to the taxation of robots could be interpreted to hold for a large entity such as all OECD countries taken together.

From a more technical perspective, we can state the following result that follows from the effects of a robot tax on the level of income.

Proposition 3. *The canonical OLG model with automation and without a robot tax does not feature transitional dynamics. Introducing a robot tax re-introduces transitional dynamics.*

Proof. From the inspection of Equation (14), we know that there are no transitional dynamics in the baseline model. Observing Equation (29), there are transitional dynamics and the economy reaches a steady state if the multiplicative term of k_t is smaller than one. Rearranging, the multiplicative term of k_t is given by

$$\frac{1}{1+n}\frac{\beta}{1+\beta}\frac{\tau}{1+\alpha\tau}(1-\alpha)^{2-\alpha}\alpha^{\alpha}(1+\tau)^{\alpha-1} < 1. \tag{32}$$

This establishes the proof for Proposition 3. □

Note that a comparison of condition (32) to k^{τ} given by (30) makes clear that condition (32) is always satisfied in an interior equilibrium, i.e., an equilibrium in which $k^{\tau} > 0$.

Finally, we would like to stress an important point. As is intuitively clear, the introduction of more traditional tax-subsidization schemes, for example, taxes on both types of capital or on labor, would not alter the stagnation result (Proposition 1) because these types of taxes would even be less effective than a robot tax in fostering saving by the young workers.

6 Conclusions

We demonstrate that the canonical OLG model of Diamond (1965) implies economic stagnation even in the face of automation. This holds true despite the overall production structure resembling the properties of an AK growth model without the diminishing marginal product of physical capital that is responsible for the well-known convergence mechanism toward a steady-state equilibrium. The reason for stagnation is that, in this framework, households exclusively save out of their labor income. By definition, however, automation competes with labor and depresses the wage rate. This reduces the saving and investment potential of households and prevents the economy from growing. Our results explain the numerical findings of Sachs and Kotlikoff (2012), Benzell et al. (2015), and Sachs et al. (2015) in the simplest analytically tractable setting. However, the results also illustrate that the phenomenon of stagnation in the presence of automation is not generalizable to other models of capital accumulation in which households re-invest a fraction of their asset incomes.

We analyze the effects of a robot tax in this setting and show that it has the potential to raise per capita capital and per capita output at the steady state. However, it cannot overcome the stagnation equilibrium of the economy. Furthermore, the baseline OLG model with automation does not feature transitional dynamics, whereas a robot tax re-introduces transitional dynamics into the model. From a policy perspective, the successful implementation of a robot tax is only feasible if it is introduced by many countries because of the possibility that capital moves to jurisdictions in which there is no robot tax. This calls for a strong international collaboration when considering the introduction of robot taxes.

In the end we would like to stress once more that our central results provide a cautionary tale in the debate on automation. The reason is that the underlying modeling structure of saving/investment decisions matters more in assessing the effects of automation than in the analysis of other phenomena in the growth literature. Intergenerational frictions open a channel thought which automation potentially has a negative effect on economic growth. However, our results should not be misinterpreted as a contribution showing that automation necessarily implies stagnation.

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