A Motivation

Policy interaction in this economy also involves responses of the policy instrument of one institution to an exogenous shock to the instrument of the other institution. Thus, it is quite natural to ask, how these exogenous policy shocks propagate through the economy under certain policy regimes, once heterogeneous expectations are present. In particular, we examine how the economy responds to a transitory, contractionary monetary, or, a negative fiscal policy shock under the different policy regimes in which determinacy prevails. For this purpose, we present simulated impulse response functions (IRFs) in our baseline model with monetary policy rule (10). Type $\varsigma = 1$ agents have RE. Expectational heterogeneity is examined by either varying χ for given ι or vice versa.

B Calibration

The calibration is guided by the estimates and the approach of Davig and Leeper (2006, 2011).[1] Note that the estimates for α, γ_0 and γ are from a specification corresponding to the case $\chi = 1$, i.e., the RE benchmark. The choice of β puts the simulation in a quarterly context and implies a quarterly real interest rate of 1%. Next, $\sigma_1 = 1$ implies logarithmic preferences over consumption, and $\sigma_2 = 2.6$ is based on Mankiw and Summers (1986). $\mathcal{A}$ is set to match the average US monetary base velocity in the deterministic steady-state of 2.4 defined by (cP/M), see Davig and Leeper (2011, p.218). Deterministic steady-state output, y, is normalized to unity and government purchases, g, account for 20% of it. The steady-state gross inflation rate, π, is chosen to correspond to an average of 2%. The steady-state values for R, m, b, and $\tilde{C}$ are calculated as outlined in Evans and Honkapohja (2007, p.688).

Parameter	α^{PM}	α^{AM}	γ_0^{PF}	γ_0^{AF}	γ^{PF}	γ^{AF}	β	σ_1	σ_1	y	g	π
Value	0.5305	1.2936	0.029	0.004	0.071	-0.025	0.99	1.00	2.60	1.00	0.20	1.02

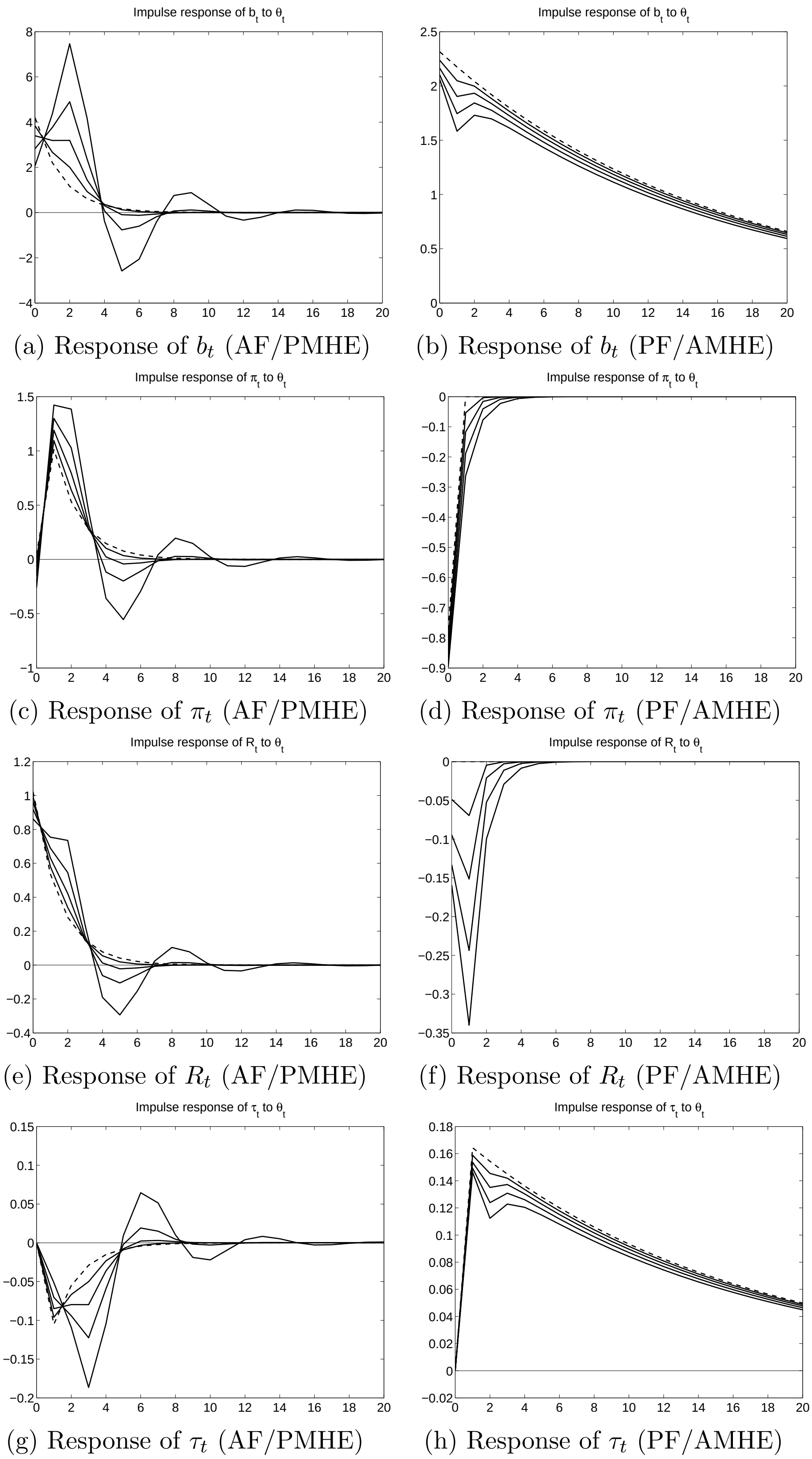
Table 1: Calibration

C Simulated Impulse Responses to a Transitory Monetary Policy Shock θ_t

First, we consider a contractionary monetary policy shock. Figure 1 depicts the simulated IRFs of the endogenous variables to a contractionary one standard deviation transitory monetary policy shock, θ_t, for the calibration in Table 1.[2] The panels in the left column are for the AF/PMHE regime, the ones in the right column are for the PF/AMHE regime.

[1] Evans and Honkapohja (2005) use a different calibration for the very same model, which is explicitly not chosen to match the data.

[2] We restrict the analysis to transitory shocks on purpose. It allows us to illustrate the persistence generated by heterogeneous expectations.



(a) Response of b_t (AF/PMHE) (b) Response of b_t (PF/AMHE)

(c) Response of π_t (AF/PMHE) (d) Response of π_t (PF/AMHE)

(e) Response of R_t (AF/PMHE) (f) Response of R_t (PF/AMHE)

(g) Response of τ_t (AF/PMHE) (h) Response of τ_t (PF/AMHE)

Figure 1: IRFs to a contractionary one standard deviation transitory *monetary* policy shock, θ_t, under both regimes. Expectations are calibrated to $\iota = 0.9$ with the solide lines representing the cases $\chi \in \{0.6, ..., 0.9\}$. The dashed line is for the RE benchmark case $\chi = 1$.

In short, comparing the simulated IRFs to a contractionary one standard deviation transitory monetary policy shock, θ_t, under $\chi = 1$ to $\chi < 1$ yields the following insights. (I) For both, the PF/AMHE and AF/PMHE regime, the impact effects on π_t (and consequently for the other variables) have higher magnitude if $\chi < 1$ and may also have the opposite sign. (II) Given $\chi < 1$, the IRFs are persistent, even though the shock is transitory. Persistence is higher in the AF/PMHE regime and generally increases with decreasing χ. (III) We observe monotonic convergence for $\chi = 1$ and for $\chi < 1$ under the PF/AMHE regime. However, under AF/PMHE the most striking feature are hump-shaped responses, undershooting of the steady-state and dampening oscillations if $\chi < 1$. The amplitude of the latter increases with decreasing χ.

A careful inspection of (13) under the respective policy regime provides the intuition for the aforementioned observations. Notice that, given our calibration, $K_1 \geq 0$, $K_2 < 0$, and $K_3 \geq 0$ as well as $\partial K_1/\partial\chi < 0$, $\partial K_2/\partial\chi > 0$, and $\partial K_3/\partial\chi < 0$ for $K_i \neq 0$ ($i \in \{1, 2, 3\}$). Now, in case of the PF/AMHE regime restriction (ii) (on p.12) applies. Therefore the coefficient K_2 yields a decrease in π_t on impact for $\chi = 1$ and an even stronger decrease on impact, if $\chi < 1$, as one can observe in Panel 1d. While $\chi = 1$ implies $K_3 = 0$, this coefficient increases, when χ decreases and in this way π_{t-1} generates the persistence in the IRFs. Thus, expectational heterogeneity ultimately causes persistent responses to a transitory monetary policy shock.

Consequently, R_t does not respond in case of $\chi = 1$ as one can see from Panel 1f. The reason is that feedback to π_t perfectly offsets the direct effect of the shock on R_t. In case of $\chi < 1$, the initial drop in π_t is larger and the feedback to this drop outweighs the direct effect of θ_t on R_t. Likewise, π_t responds persistently and so does R_t, while giving feedback to the former.

The impact effects on π_t and R_t in turn explain the responses of b_t on impact in Panel 1b. The more negative is the net effect of the former two variables, the smaller is the response of b_t on impact. While π_t returns to the steady-state, b_t decreases further. Together with the lagged response of τ_t to b_t, the responses of π_t and R_t also support the relatively slow convergence of b_t.

For the AF/PMHE regime restriction (i) on p.12 holds. Now consider the case of $\chi = 1$. Given the characteristics of the coefficients K_i, the monetary policy shock tends to decrease π_t on impact, but this direct negative effect on π_t is outweighed by the positive effect of an increase in b_t on impact as Panel 1c illustrates. Furthermore, Panel 1e exhibits the passive policy feedback to π_t due to PMHE. In consequence R_t increases on impact. Intuitively, these responses are also consistent with the substitution effect described in the paper.

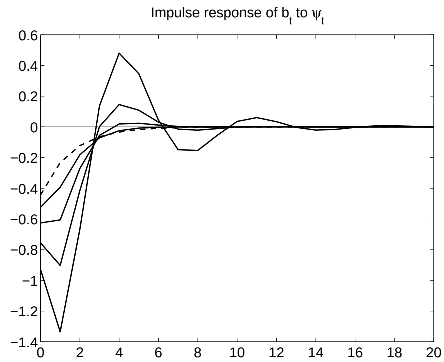
In the subsequent periods, (13) implies a persistent convergence of π_t from above, as there is a positive effect of current b_t. R_t follows. Moreover, τ_t in Panel 1g shows a lagged response to the increase b_t. However, it is in opposite direction compared to the PF/AMHE regime, which is consistent with our calibration of AF. This and the adjustment in π_t drive b_t back to the steady-state.

For $\chi < 1$, Panel 1c illustrates an amplified impact effect on π_t. Now π_t decreases slightly on impact, consistently with a more negative K_2 and a less positive K_1. The extent of amplification on impact also determines the extent to which the responses of R_t and b_t are mitigated on impact.

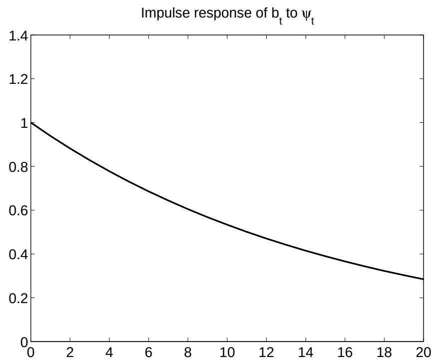
Following the impact effects, next to the positive effect of current b_t heterogeneous expectations also imply a negative effect of lagged π_t. If χ is small enough, e.g., $\chi = 0.6$, the latter effect creates a hump, undershooting of the steady-state or even dampening oscillations in π_t. These dynamics of π_t in turn lead the oscillations in b_t, R_t, and τ_t through exactly the same channels as discussed for the case of $\chi = 1$. Thus, the ultimate source of the hump, undershooting and oscillations is the expectational heterogeneity, as it triggers an interplay between b_t and π_{t-1} in the inflation dynamics.

D Simulated Impulse Responses to a Transitory Fiscal Policy Shock ψ_t

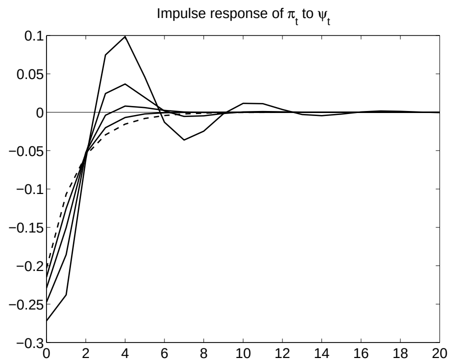
Next, Figure 2 below exhibits the simulated IRFs to a negative one standard deviation transitory fiscal policy shock, ψ_t, for the calibration in Table 1. The simulated IRFs indicate that (IV) the impact effects on π_t are identical and independent of χ under the PF/AMHE regime. Notice that π_t and R_t are unaffected by ψ_t by construction of the monetarist solution. Only τ_t responds to a rise in b_t. The opposite is indeed true under the AF/PMHE regime, where π_t is affected by b_t. (V) Again, $\chi < 1$ generates persistent responses to a transitory shock and (VI) convergence has similar characteristics as in the case of θ_t. We find monotonic convergence for $\chi = 1$ and the same for $\chi < 1$ in case of PF/AMHE, whereas under AF/PMHE we observe dampening oscillations if $\chi < 1$.



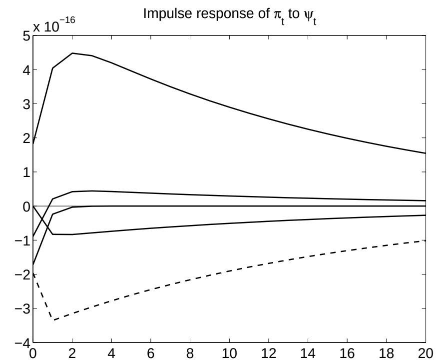
(a) Response of b_t (AF/PMHE)



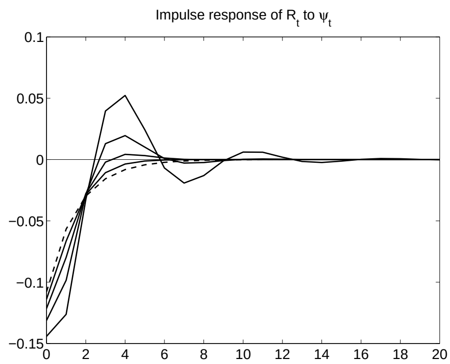
(b) Response of b_t (PF/AMHE)



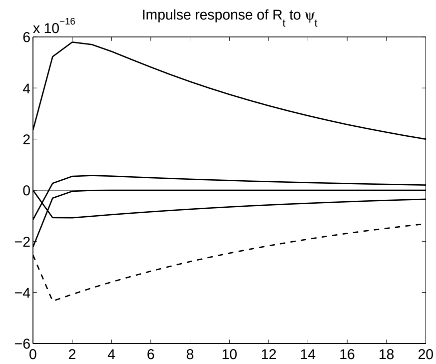
(c) Response of π_t (AF/PMHE)



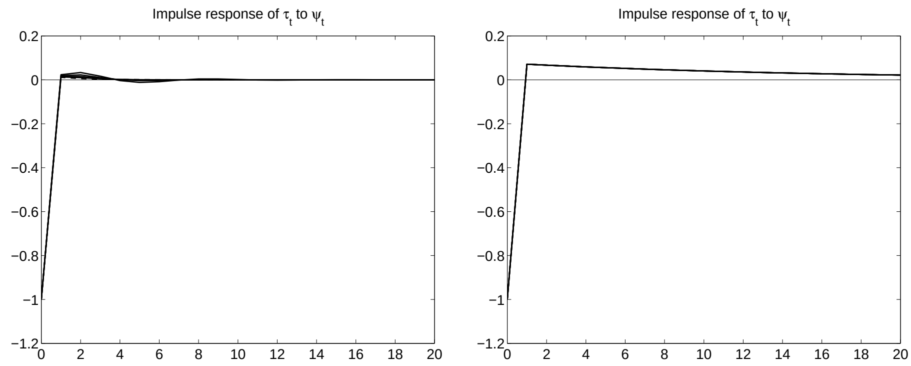
(d) Response of π_t (PF/AMHE)



(e) Response of R_t (AF/PMHE)



(f) Response of R_t (PF/AMHE)



(g) Response of τ_t (AF/PMHE) (h) Response of τ_t (PF/AMHE)

Figure 2: IRFs to a negative one standard deviation transitory *fiscal* policy shock, ψ_t, under both regimes. Expectations are calibrated to $\iota = 0.9$ with the solide lines representing the cases $\chi \in \{0.6, ..., 0.9\}$. The dashed line is for the RE benchmark case $\chi = 1$.

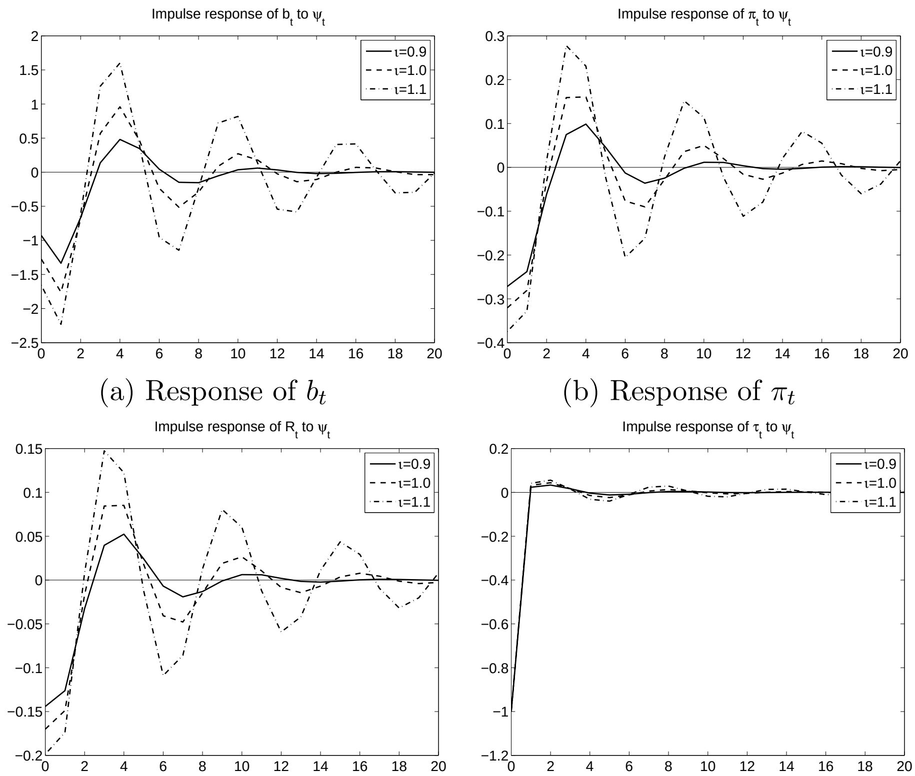
Again, examining (13) helps to clarify these observations. Clearly π_t cannot be affected by a fiscal policy shock in case of the PF/AMHE regime, as coefficient $K_1 = 0$ and the other terms are irrelevant (recall restriction (ii)). This applies for $\chi \leq 1$ and explains, why π_t, and consequently R_t, do not respond at all. As π_t does not respond to the shock, b_t rather mechanically rises due to the transitory drop in τ_t, as one can spot in Panels 2b and 2h. In the subsequent periods, τ_t is raised, as PF feeds back sufficiently strong to offset the rise in b_t.

In case of the AF/PMHE regime, restriction (i), the negative fiscal policy shock from Panel 2g causes b_t to decrease on impact, which also triggers a drop in π_t via (13) and a negative response of R_t on impact. Under $\chi = 1$ monotonic convergence towards the steady-state follows, as $K_3 = 0$. Moreover, τ_t shows (almost) no response to b_t below its steady-state due to the AF stance. In contrast, $\chi < 1$, and in consequence $K_3 > 0$, make π_t dependent on its lagged value during the transition. The latter once more creates reciprocity between b_t and π_{t-1}, which potentially once more generates dampening oscillations in π_t, which are then followed by b_t and R_t.

Why does b_t fall on impact? Because the negative tax shock creates a positive income effect at the beginning of the period. Thus, households will want to save less, and government can issue less bonds. This is reflected in a negative deviation of end-of-period b_t. Is this consistent with the IRFs under the PF/AMHE regime? Indeed; in contrast to the AF stance, households anticipate that there will be above steady-state taxes in the future under PF. Thus, there is a negative wealth effect under the PF stance. Hence, the government can partially offset the temporary drop in τ_t by issuing more bonds, as households want to save more.

E Robustness regarding ι

Finally, Figure 3 below shows variations of ι for given χ. It becomes evident that the dampening oscillations under the AF/PMHE regime emerge because of $\chi < 1$. Notice that the emergence of the dampening oscillations under the AF/PMHE regime is only modestly dependent on whether the coefficient ι is below or above unity. This is important, as the choice of ι usually turns out to discriminate between fundamentally different dynamics in homogeneous expectations economies. In our case, values of ι above or below one affect the amplitude and frequency of the dampening



(a) Response of b_t (b) Response of π_t

(c) Response of R_t (d) Response of τ_t

Figure 3: IRFs to a negative one standard deviation transitory *fiscal* policy shock, ψ_t under an AF/PMHE regime for $\chi = 0.6$ and $\iota \in \{0.9, 1.0, 1.1\}$.

oscillations. However they are not decisive for whether or not the oscillations occur.

F Discussion and Conclusion

Despite the rather conceptual character of this analysis, the occurrence of persistent IRFs, in our *Neo-Classical model*, especially to monetary policy shocks, is worthwhile. It states an example of how persistence can be a feature of an economic model, which does not rely on nominal rigidities.[3] Anufriev et al. (2013) carry out non-linear stochastic simulations in their frictionless model and also find high persistence. These results are of particular interest in light of the debate on the plausibility of nominal rigidities, see, e.g., De Grauwe (2010, 2011, 2012) and others.

The predictions of the model are also related to a literature that suggests expectation formation as a main source of macroeconomic persistence and business cycle amplification. For instance, Milani (2007) estimates a homogeneous expectations DSGE model, including sticky prices, habit formation, and inflation indexation, under the assumption that agents are adaptive learners. This assumption introduces systematic forecast errors that *vanish asymptotically*. It turns out that the data clearly points to adaptive learning as the key driver of macroeconomic persistence assigning a minor role to nominal rigidities. In our model, expectation formation of agents with AE, i.e., *persistent* forecast errors, drives macroeconomic persistence.

Likewise the experimental evidence provided by Adam (2007) documents that many subjects

[3] Our finding is in line with Zhao (2007).

deviate from RE when forecasting aggregate variables. A Restricted Perceptions Equilibrium[4] (RPE) can emerge, in which agents use (misspecified) univariate forecast for π_t. Such forecasts are similar to the ones of the AE in our model. Adam (2007) finds that in such a RPE the IRFs to aggregate shocks have similar properties as in our simulations.[5]

Furthermore, the demonstrated business cycle amplification and higher volatility due to the presence of heterogeneous expectations appears to be a key characteristic of infinite-horizon models, see for instance Branch and McGough (2011) or Kurz et al. (2013).

Finally, the hump, undershooting, and dampening oscillations in the simulated IRFs to fiscal and monetary policy shocks under the AF/PMHE regime, indicate that heterogeneous expectations might be one potential source to explain business cycles. Farmer (1999, p.141ff.), Farmer and Guo (1994, p.67ff.), and Azariadis et al. (2004, p.336ff.) document the empirical relevance of oscillating responses to shocks and give reference to further evidence. These studies also discuss homogeneous RE models that predict such oscillations. Under homogeneous adaptive learning, Mitra et al. (2013) and Gasteiger and Zhang (2014) find this prediction for *permanent* fiscal policy changes in a RBC and a Ramsey model respectively. In contrast, in our model these empirically relevant cyclical dynamics emerge for *transitory* shocks, caused by expectational heterogeneity.

In sum, the IRFs to both shocks appear to have three striking characteristics. First, in contrast to the homogeneous RE benchmark case, the IRFs of inflation under heterogeneous expectations exhibit significant persistence. This feature is absent from the benchmark RE *Neo-Classical* model, when monetary policy shocks occur. The lack of persistence was one of the main motivations for the introduction of nominal rigidities over the last decades, and is not an uncontroversial issue in the profession. In our model nominal rigidities to generate persistence are obsolete, as long as there is reasonable heterogeneity. Second, under heterogeneous expectations, the impact effects can have higher magnitude and the opposite sign compared to the RE benchmark, as expectational heterogeneity not only introduces lagged inflation to the inflation dynamics, but also amplifies the influence of fiscal variables. Finally, convergence can occur in dampening oscillations, a well documented feature in US post-war data. Such interesting business cycles dynamics are uncommon to many homogeneous expectations models, but an intrinsic feature of our model.

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[4] See for example Evans and Honkapohja (2001).

[5] In addition, recent research on survey expectations by Fuhrer (2015a,b) suggests the persistence inherent in private sector expectations as an important source of macroeconomic persistence. Fuhrer (2015b) also develops a structural DSGE model with survey expectations based on similar assumptions as the model proposed herein.

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